Experiments on power generation from a moving plasma

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Electrical power of 0·32 MW has been extracted for a period of 100 μsec from a plasma of shock-ionized argon travelling at 4×10^5 cm per sec through a magnetic field of 10,000 G. Currents of more than 10,000 amp are drawn and the resulting modification of the applied field is considered.

Maximum power is obtained when the external load is matched to the plasma generator which has an internal impedance equivalent to its own electrical resistance in series with a resistance arising from its behaviour as a compressible fluid.

Values of the electrical conductivity of the plasma obtained in these experiments (about 3×10^3 mho per metre) show that the plasma resistance is controlled by electron motion and these values are in good agreement with those found by other methods.

Introduction

Experiments on the generation of electrical power by converting the energy of a fast-moving electrically conducting fluid are now being carried out in several laboratories. Pilot plants now operating employ the continuous flow system where the gas has a temperature in the region of 2000 °K and its conducting properties are enhanced by the introduction of easily ionized substances such as potassium. A report on some of the experiments at Avco, U.S.A., has been made by Rosa (1960). A theoretical analysis of the problem has recently appeared in this journal (Neuringer 1960).

The principle of all of these experiments is the same. A fast-moving plasma crosses a region of magnetic field the lines of which are transverse to its flow direction. If the flow velocity is u cm per sec and the field strength is B G an electric field $10^{-8}(\mathbf{u} \times \mathbf{B})$ V per cm is developed in the plasma. Currents of density $j = \sigma u B$, where σ is the electrical conductivity of the plasma, will flow when a short-circuit path is available. Such a path is provided by completing an external circuit via a pair of electrodes whose faces are parallel to the flow direction and to the lines of the magnetic field (figure 1).

In the experiments reported here the process is essentially a pulsed system and not a continuous flow method. Power is extracted from the plasma for periods of about 100 μsec which is the duration of plasma flow through the electrode and the magnetic field system. The flow velocity is about 4×10^5 cm

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per sec and the field strength is 10,000 G so that the electric field across the plasma is about 40 V per cm.

Earlier experiments of a similar type were presented in a thesis by Brogan (1956) who considered in some detail the resistive effect of the cold boundary layer between the hot plasma and the cold electrodes. In the present experiments, boundary-layer effects appear to intrude only when the currents drawn

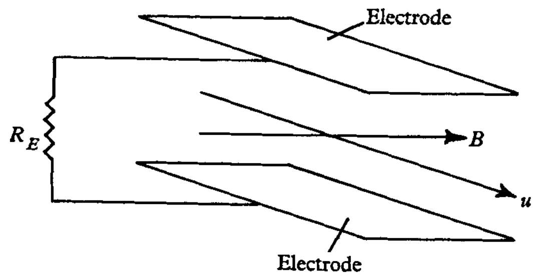

FIGURE 1. Disposition of the electrodes with respect to the plasma flow (velocity u) and the magnetic field (B). An electric field $\mathbf{u} \times \mathbf{B}$ generates a current which flows through the circuit closed by the external load R_E.

fall below 100 amp per cm^2 of electrode surface. At larger currents the resistance presented by the plasma between the electrodes leads to a value of the electrical conductivity which is in good agreement with results obtained by other methods.

This electrical conductivity is controlled by the motion of the electrons. In a recently published paper by Sakuntala, von Engel & Fowler (1960) a similar experimental arrangement is described; this yields values of the electrical conductivity which are governed by ionic mobility. The discussion at the end of this paper suggests that there is no discrepancy between these results, but that the significant conductivity depends upon the magnitude of the current flowing through the plasma.

Experimental conditions

The plasma is generated as a column of shock-ionized argon in a combustion-driven shock-tube, details of which are given in Pain & Smy (1960*a*). The length of the plasma column varies between 5 and 50 cm depending upon the initial conditions in the shock tube. The plasma has a velocity of 4×10^5 cm per sec, a temperature near 12,000 °K and a degree of ionization, α, of about 20 %.

It passes through a Pyrex glass tube 30 cm long, of 5 cm internal diameter and at the mid-point of which are located the magnetic field and the electrode system. The magnetic field is transverse to the plasma flow and is formed by two spirally wound coils arranged as a Helmholtz pair, one on each side of the shock tube.

Pulsed magnetic fields of more than 10,000 G, uniform across the shock tube are developed when a condenser bank of 900 μF, rated at 5 kV, is discharged through the coils. The period of discharge is approximately 1 msec and the plasma is timed to pass through the region when the field is at its maximum. The flow duration is much less than 1 msec so that a constant field is seen by the plasma.

The electrode system is formed by a pair of conductors set in the walls of the Pyrex tube and machined flush with the inner surface.

The directions of plasma flow, of magnetic field and of the normal to the two electrode faces form a set of three mutually perpendicular axes as shown in figure 1.

Small electrode system

The first pair of electrodes was designed to carry out a preliminary investigation on the response of such systems. Each electrode was a 2 cm length of copper rod of circular cross-section 1 cm in diameter.

External resistances varying from 100 KΩ to 0·01 Ω were wired in symmetric array close to the glass walls of the tube to keep the external inductance to a

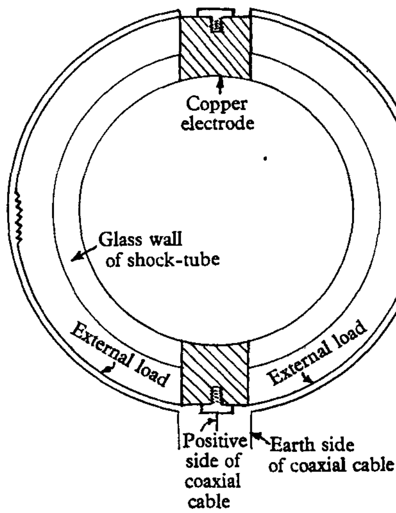

FIGURE 2. The small copper electrodes are inserted as studs in the glass walls of the shock-tube. The external load resistance is arranged symmetrically around the tube, very close to the tube wall and to the lead which connects one electrode to the earth side of a coaxial cable. The positive side of the cable feeds the signal from the other electrode to the C.R.O.

minimum. Electrical pick-up was also minimized by the voltage-measuring device shown in figure 2. The voltage developed across the external resistance R_E was recorded on a cathode-ray oscilloscope and typical traces obtained with initial shock-tube argon pressures of 10, 1 and 0·1 mm of mercury are shown in figure 3.

The recorded voltage allows the induced current and the power dissipated in the external load R_E to be calculated and the electrical conductivity of the plasma to be found. The small external resistances used in this experiment were made from resistance wire. The electrical energy dissipated in these resistances caused a negligible change in their value and their dimensions were such that the ohmic resistance was independent of frequency over the range of frequencies involved in the induced current pulse.

If R_p is the resistance of the plasma between the electrodes, R_E the external resistance, V_E the measured voltage across R_E, d the electrode separation and B the magnetic field strength, then the equivalent circuit yields the relation

$$V_E = uBdR_E/(R_E+R_p),$$

that is

$$\frac{1}{V_E} = \frac{1}{uBd} + \frac{1}{uBd}\frac{R_p}{R_E}. \tag{1}$$

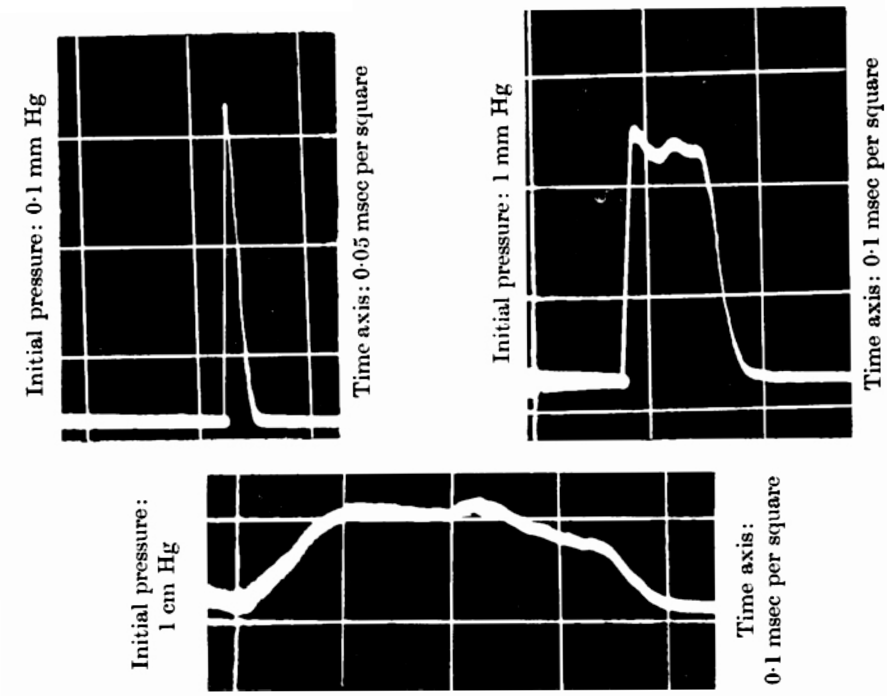

FIGURE 3. Typical voltage traces obtained with the small electrodes for initial channel pressures of 1 cm, 1 mm, and 0·1 mm of mercury. The rise and decay of the 1 cm trace is attributed to the variation of the applied field of half-period equal to the flow duration.

Except where stated all experiments were carried out with an initial argon pressure of 1 mm of mercury in the shock-tube.

The plasma resistance R_p is not simply $d/(A\sigma)$, where A is the electrode area and σ is the electrical conductivity of the plasma. This formula is valid only when $A \gg d^2$. In the present case $A \ll d^2$, so that electrode end-effects must be considered.

The effective resistance presented by the plasma between the electrodes was found by filling the space between the electrodes with a salt solution of known concentration and electrical conductivity σ'. The resistance R' of this solution between the electrodes was then found using an alternating current source and the relation $R_p = R'\sigma'/\sigma$ effectively determines R_p. Keeping the plasma flow conditions constant a series of measurements of V_E was made for different values of R_E. If equation (1) were valid then the graph of $1/V_E$ *vs* $1/R_E$ should be a straight line with a slope $R_p/(uBd)$. For values of R_E less than 1 Ω there is a gradual departure from the linear relationship as shown in figure 4. For R_E

greater than 1 Ω the behaviour was more complex and this is discussed in more detail in a later section. For this reason the slope of the curve in the vicinity of $R_E = 1\,\Omega$ was used to determine R_p.

The graph in figure 4 curves upwards and this may be explained qualitatively in terms of the plasma being slowed down by the braking pressure $P_z = \int(\mathbf{j} \times \mathbf{B})_z\,dz$, where z is the co-ordinate in the direction of flow and j is the induced current density. The integral is carried out over the extent of the interaction region. This braking pressure is largest at low external resistances and high induced current densities.

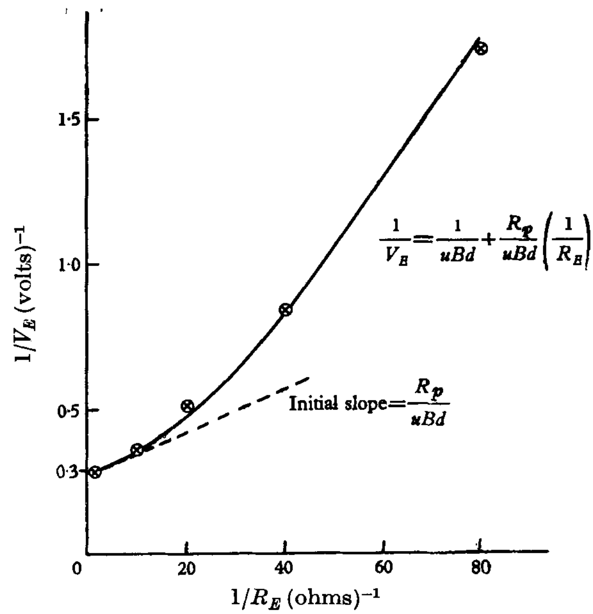

FIGURE 4. Reciprocal measured voltage V_E *vs* reciprocal external resistance R_E, for $R_E < 1\,\Omega$. The change of slope is attributed to the variation of σu, where σ is the electrical conductivity of the plasma and u is the flow velocity. For R_E close to 1 Ω the change of σu is negligible and the initial slope of the graph enables the values of σ to be calculated. Measurements recorded on this graph were made with an initial argon pressure of 1 mm of mercury.

If the graph of equation (1) were linear its slope of $R_p/(uBd)$ would equal $R'\sigma'/(\sigma uBd)$ which involves the product σu.

When the plasma slows down under the influence of the braking pressure its thermal energy (and hence its electrical conductivity) is increased at the expense of its kinetic energy. In practice the product σu is not invariant but is proportional to u^n, where n is positive and less than unity. This may be seen from the following argument, which assumes a steady-state condition with small Joule dissipation.

Mass conservation requires ρu to be constant, where ρ is the plasma density. This, with the adiabatic condition $p/\rho^\gamma =$ const. and the equation of state $p/\rho = RT$, yields $RTu^{\gamma-1} =$ const. If the temperature dependence of the conductivity follows the $T^{\frac{3}{2}}$ law, then $\sigma u \propto u^{\frac{1}{2}(5-3\gamma)}$, i.e. σu is a constant for $\gamma = \frac{5}{3}$. However, the value of γ for the plasma is nearer 1·1 than 1·66—Patrick & Brogan (1959)—and $\sigma u \propto u^{0\cdot85}$.

As $1/R_E \rightarrow \infty$ the slope of the graph in figure 4 tends to about twice the value it has as $1/R_E \rightarrow 0$, i.e. at large current densities the observed value of σu tends to half its original value upon entering the electrode system. The current densities flowing through the small electrodes are insufficient to modify the applied field B seriously, but in the case of the large electrodes the applied field is distorted; this factor is considered later.

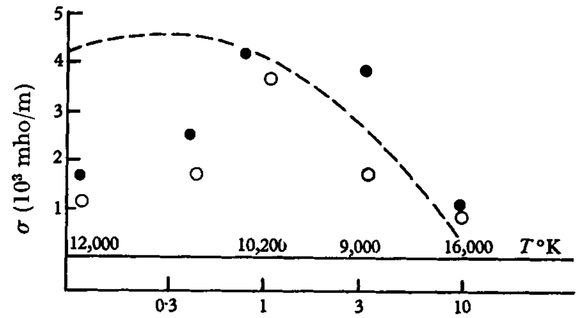

FIGURE 5. Values of the electrical conductivity of the plasma for different initial conditions in the shock-tube. ○, Experimental results; ●, results found by Pain & Smy (1960*b*). The dotted curve shows the theoretical values computed by de Leeuw (1958) using the method of Lin *et al.* (1955).

The slope of the graph as $1/R_E \rightarrow 0$, i.e. at low current densities, allows the value of the electrical conductivity of the plasma to be calculated. Measurements at low current densities were made for different plasma flow conditions, that is for different initial shock-tube pressures, and the electrical conductivity of the plasma was found in each case. These values are shown in figure 5 where the dotted curve shows the theoretical values derived by de Leeuw (1958) using the theory of Spitzer & Harm modified by Lin, Resler & Kantrowitz (1955).

These values agree quite well with results obtained by Lin *et al.* (1955) and Pain & Smy (1960*b*) using other experimental techniques.

The possibility that the external resistance is short circuited by a return current path in the plasma or the boundary layer may be discounted for the following reasons.

The magnetic field has a value of more than 5000 G over a region of 10 cm in the flow direction and a value of 10,000 G over a length of 7 cm. The electric field $\mathbf{u} \times \mathbf{B}$ is therefore unidirectional for some distance on either side of the electrodes and any current path through the plasma would be long enough to exceed the resistance of the external load. The resistance of a path through the cold boundary layer is also greater than the external load.

Large electrode system

The small pair of copper electrodes was replaced by a large pair of brass electrodes 7 cm long and 4 cm arc. The cross-section of these electrodes is shown in figure 6 which demonstrates this arrangement as the best approximation, using a circular cross-section, to the ideal case of a square cross-section with one pair of opposite faces forming parallel electrodes. When the shock tube has a square cross-section the problem may be treated by one-dimensional analysis.

The pulsed magnetic field perpendicular to the flow has a frequency which requires the shielding effect of the brass electrodes to be considered. Search coil measurements showed that the change in the total flux across the shock-tube in the vicinity of the electrodes was less than 5 %.

The external resistances used with these electrodes varied from 0·01 to 0·00025 Ω and were constructed from resistance wire or copper strip. The dimensions of these resistances were chosen to reduce temperature change and skin effects to a minimum.

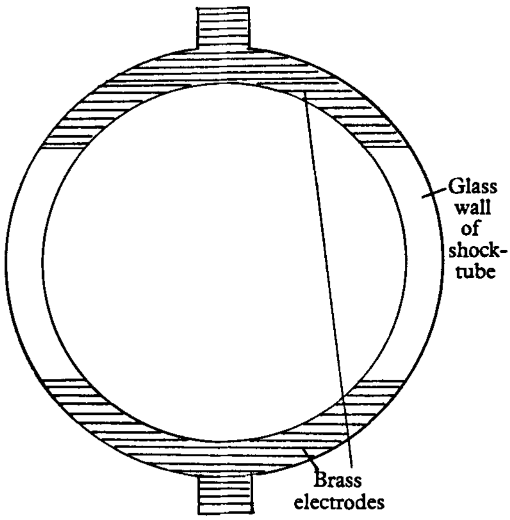

FIGURE 6. Cross-section of the large brass electrodes of 7 cm length and 4 cm arc. The external resistance arrangement is the same as in figure 2.

The resistance of the external load was found by measuring the voltage across the resistances when it was conducting 10 amp and all resistance values were measured *in situ*.

No trouble with contact resistances was experienced when the metal contacts were clean and held together under strong pressure. The voltage V_E across the external load was measured in the same way as with the small electrode system.

Despite great care with the arrangement of the external load the resulting voltage wave-form showed considerable pick-up because of the large induced current flowing in the external load.

The ohmic voltage was found from the oscilloscope trace by taking the average voltage over the duration of the wave-form, i.e. the ohmic voltage V_0 is given by $\int V\,dt/\tau$, where τ is the pulse duration. The superimposed pick-up voltage V_p is thus eliminated because its integral over all time,

$$\int V_p\,dt \propto \int_{t=0}^{t=\infty} \frac{di}{dt}\,dt,$$

is zero.

The voltage oscillograms were reproducible from run to run. The magnetic field value was 10,000 G and all measurements were made with an initial pressure of argon in the shock-tube equal to 1 mm of mercury.

Measurements with the large electrode system

Given an electrode area of $28\,\text{cm}^2$ and a current density $j = \sigma u B$, then the maximum total current I_m as $R_E \to 0$ should be $28\sigma u B$ if σu is considered invariant. In table 1 the measured value of I_m is seen to be 14,000 amp so that the average value $\overline{\sigma u} = 5 \times 10^6$ m.k.s. units.

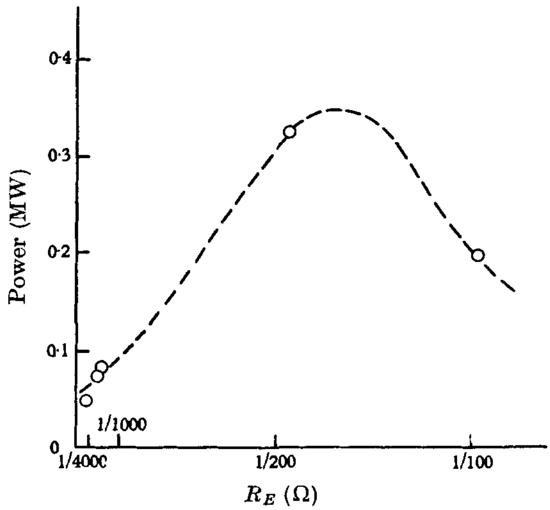

FIGURE 7. Power extracted from the plasma *vs* external load. The maximum power extracted occurs for a value of R_E of about 1/150 Ω. This matches the internal impedance of the plasma generator when it includes a resistance which arises from its behaviour as a compressible fluid.

The plasma flow conditions upon entering the electrode system yield $\sigma = 3 \times 10^3$ mho per m and $u = 4 \times 10^3$ m per sec so that the average value $\overline{\sigma u}$ is about half the original value upon entry and σu is not invariant.

The reduction of the flow velocity may be seen as a decrease in the voltage available for power generation. This may be expressed in terms of an extra resistance in series with the internal impedance R_p of the plasma generator, which represents the electrical impedance developed when the plasma is slowed down by the interaction between the induced current and the applied field. This extra resistance is a non-linear term which depends upon the strength of interaction, so that the plasma generator may have a variable internal impedance.

Writing the extra resistance as R, then as $R_E \to 0$,

$$\frac{R_p}{R+R_p} = \left\{\frac{\overline{\sigma u}\ \text{(measured)}}{\sigma u\ \text{(calculated)}}\right\} \sim \tfrac{1}{2},$$

so that
$$R \sim R_p = \frac{d}{A\sigma} = \frac{1}{300}\,\Omega.$$

In figure 7, where power extraction is plotted against the external load R_E, the maximum of the curve occurs in the region $R_E \sim 1/150\,\Omega$, where the external load matches the total impedance of the generator.

The value of the current is found directly from the voltage trace and an error of up to 15 % is probable.

R_E (Ω)	I (amp)	I^2R (W $\times 10^5$)
1/100	4,500	2
1/200	8,000	3·2
1/1800	11,400	0·7
1/2000	10,000	0·5
1/4000	14,000	0·49

TABLE 1

The arguments against short circuiting through the plasma are similar to those for the small electrodes. In this case, moreover, the existence of two currents of about 10,000 amp flowing in opposite directions in the plasma would generate interaction forces with the magnetic field acting in reverse directions. This would break up the plasma flow, an effect which has not been observed.

Modification of applied field by induced currents

When induced currents of the order of 10^4 amp flow, the induced field in the plasma may be as large as the applied field of 10,000 G and the distribution of the applied field is modified. In figure 8, z is the direction of plasma flow, the applied field is constant and in the y-direction, and the induced current is in the negative x-direction, its density j_x being equal to $\sigma u B$. The applied field is B_y/μ_0 and the induced field in the y-direction is B'/μ_0 so that the total field $B/\mu_0 = (B'+B_y)/\mu_0$; μ_0 is the permeability of free space. B' opposes B_y at z_1 in figure 8 and reinforces it at z_2. We have that

$$\mu_0 j_x = \frac{\partial B'}{\partial z} = \frac{\partial B}{\partial z}, \quad \text{since } B_y \text{ is constant}$$

and
$$\sigma u \mu_0 B = \frac{\partial B}{\partial z}.$$

Now $\int \mathbf{B}.d\mathbf{l} = \mu_0 I$, where the line integral is taken around the shaded area in figure 8 and I is the total current flowing across this area so that

$$B(z_2) - B(z_1) = \mu_0 j,$$

where j is the induced current density per cm² along the y-axis and $B(z_2)$ is the value of the field in the y-direction at z_2. By symmetry the induced field B' is equal and opposite on either side of the current sheet so that

$$B(z_1) + B(z_2) = 2B_y.$$

The braking pressure
$$P_z = \int_{z_1}^{z_2} (\mathbf{j} \times \mathbf{B})_z \, dz$$

and from
$$dz = dB/\mu_0 j \tag{2}$$

we get

$$P_z = \tfrac{1}{2}[B^2(z_2) - B^2(z_1)]/\mu_0 = \tfrac{1}{2}[B(z_2) - B(z_1)]\,[B(z_2) + B(z_1)]/\mu_0 = IB_y.$$

When σu is invariant between z_1 and z_2 then (2) integrates to give

$$B(z_2) = B(z_1)\,e^{\mu_0 u\sigma(z_2 - z_1)} = B(z_1)\,e^{z/10},$$

when the appropriate values are inserted and z is measured in cm. The applied field is thus modified as shown in figure 9.

When $B(z_2)$ remains finite the limit $\mu_0 u\sigma z \to \infty$ yields $B(z_1) \to 0$, so that

$$B(z_2) = (2B_y - B(z_1)) \to 2B_y \quad \text{and} \quad P_z \to 2B_y^2/\mu_0,$$

which is the maximum pressure that the magnetic field can exert upon the plasma. When z tends to ∞ the original expression $P_z = \sigma u B_y^2 z$ also tends to ∞.

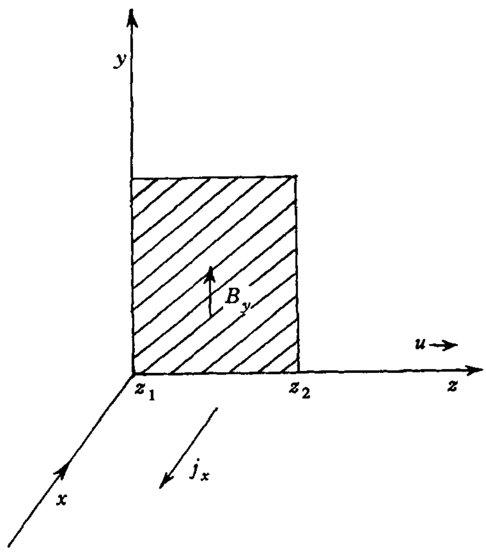

FIGURE 8. When the plasma flow velocity u is in the z-direction, and the applied field B_y is in the y-direction, the induced current j_x is in the x-direction. By considering $\int \mathbf{B}.d\mathbf{l}$ around the contour of the shaded area, the modification of B_y when j_x is large is shown not to affect the overall braking pressure, $\int_{z_1}^{z_2} j_x \times B_y\, dz$, by a large amount ($z_1$ and z_2 are the limits of the magnetic field region).

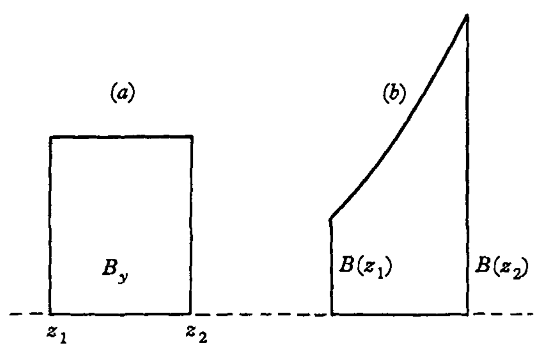

FIGURE 9. The applied field B_y extends from z_1 to z_2. When the total induced current is about 10,000 amp, the applied field is modified from (a) to (b).

Writing $\sigma u B_y^2 z$ as $(P_z)_0$ the ratio of the two expressions for P_z then becomes

$$\frac{P_z}{(P_z)_0} = \frac{[B(z_2)-B(z_1)]\,[B(z_2)+B(z_1)]}{2\mu_0 \sigma u B_y^2 z},$$

which, for $2B_y = B(z_1)+B(z_2)$, becomes

$$\frac{2}{\mu_0 \sigma u z}\left\{\frac{B(z_2)-B(z_1)}{B(z_2)+B(z_1)}\right\}.$$

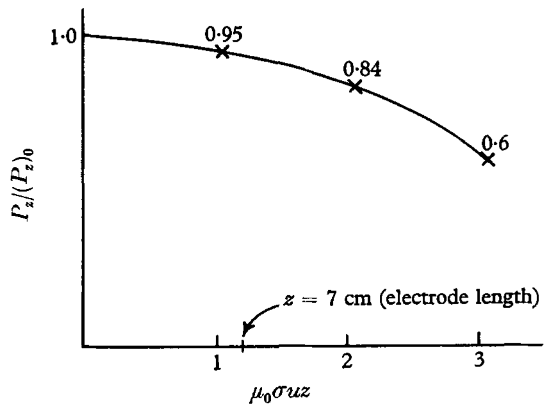

FIGURE 10. $(P_z)/(P_z)_0$ *vs* $\mu_0 \sigma u z$, when σu is considered invariant. P_z is the braking pressure developed when the magnetic field is modified by large induced currents, and $(P_z)_0$ is the braking pressure in the absence of modification. The modification for $z = 7$ cm, the length of the large electrodes, is seen to be about 5 %.

When σu is invariant, this ratio is equal to

$$\frac{2}{\mu_0 \sigma u z}\left\{\frac{e^{\mu_0 \sigma u z}-1}{e^{\mu_0 \sigma u z}+1}\right\}.$$

This expression is plotted in figure 10 with $P_z/(P_z)_0$ as the ordinate and $\mu_0 u \sigma z$ as the abscissa. With the appropriate values of $z = 7$ cm (length of large electrode) and $\mu_0 u \sigma = 0{\cdot}2$ it is seen that $P_z \sim (P_z)_0$ and that current modification of the applied field does not appreciably alter the overall braking pressure.

It should be noted that $\sigma \mu_0 u z$ is the magnetic Reynolds number so that figure 10 shows the explicit dependence upon this parameter of the reduction in braking pressure when the field is modified by induced currents.

The modification of the applied field which leads to a maximum braking pressure of $2B_y^2/\mu_0$ instead of the usual magnetohydrostatic pressure $B_y^2/2\mu_0$ also implies a corresponding change in the Lundquist number which depends on the field value B.

The significance of these modifications is greatest when the plasma is an infinite sheet of thickness $(z_2 - z_1)$ and the induced flux remains within the sheet.

Discussion

The apparent absence of boundary-layer effects when large currents flow is a feature of the experiments with the small electrodes. The Joule dissipation of a large current rapidly heats a cold boundary layer and reduces its resistance.

The various mechanisms by which a current is permitted to flow through this layer are fully discussed by Brogan (1956).

When the external load is more than 1 Ω, currents of less than 100 amp per cm^2 are drawn and the resistance through the plasma between the two electrodes is much greater than the plasma resistance alone and must include a cold boundary-layer resistance. A typical oscilloscope trace for a low current through an external load is shown in figure 11 to consist of an initial voltage pulse, much smaller than that expected in the absence of a cold boundary layer, on which is superimposed a series of peaks of very short duration which may be associated with a temporary local breakdown followed by a rapid recovery.

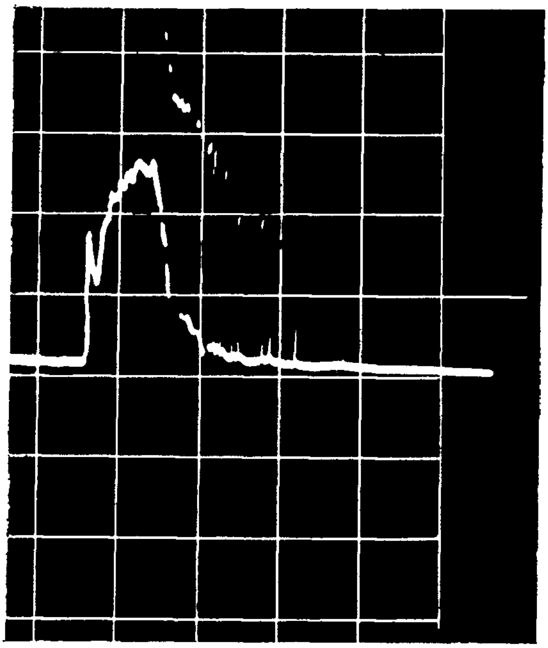

FIGURE 11. Typical oscilloscope trace obtained when current densities of less than 100 amp per cm^2 flow through the external load. The load here is 1000 Ω and the low initial voltage is considered to result from the voltage drop across a cold boundary-layer resistance. The sharp peaks are attributed to temporary breakdown of this boundary-layer resistance.

The recording of such a trace with large external loads supports the argument that no short circuiting occurs within the plasma, for short circuiting would make the shape of the trace independent of the value of the external load.

In the experiments with the large electrodes the maximum power is extracted when the external load matches the internal impedance of the generator which includes the resistance arising from the compressible nature of the plasma. This energy, 0·3 MW for 100 μsec represents about 30 % of the kinetic energy of plasma flow and must be the difference between the total energy content of the plasma at the entrance to and at the exit from the electrode system apart from heat losses to the electrodes.

Sakuntala *et al.* (1960) have used a similar experimental arrangement with hydrogen as a working fluid. Their measurements lead to a much lower value of the electrical conductivity than that expected from the motion of the electrons, and it is shown that the much lower mobility of the ions is the limiting mechanism in the flow of current. This may be true when the current is small, but need not hold when the current is large, as is shown in the following argument based on work which is to be published by J. E. Lucas of this department. Lucas has examined both theoretically and experimentally the redistribution of the electric field between two electrodes in a plasma when current is drawn. For small currents the electric field falls below the applied field $E_0 = uB$ at the anode and exceeds E_0 at the cathode because of the drift of positive ions. However, except at the anode, the field is everywhere of the order of E_0. This case is shown in figure 12 (*a*) where the current is controlled by the ions, their velocity being $v = \mu^+ E_0$, where μ^+ is the ion mobility. The condition to be satisfied is that the integral $\int E\,dl$ under the curve must be equal to $E_0 d$, where d is the electrode separation.

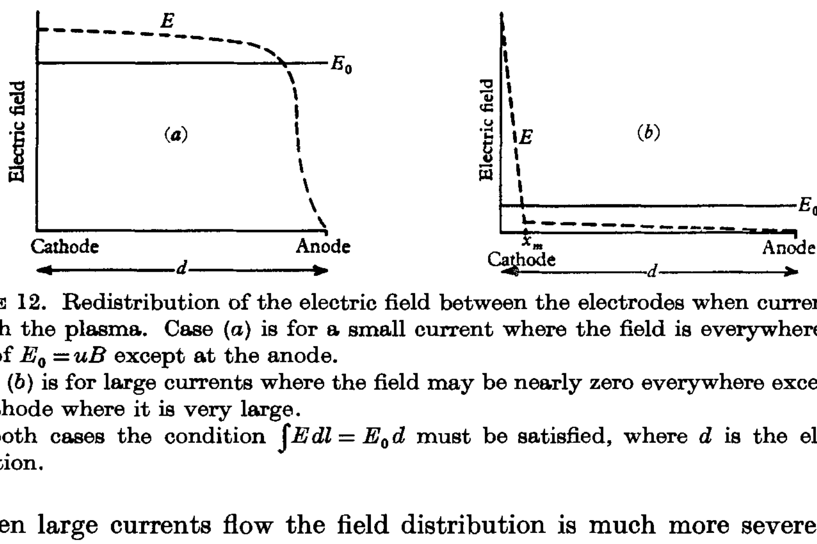

FIGURE 12. Redistribution of the electric field between the electrodes when current flows through the plasma. Case (*a*) is for a small current where the field is everywhere of the order of $E_0 = uB$ except at the anode.

Case (*b*) is for large currents where the field may be nearly zero everywhere except near the cathode where it is very large.

In both cases the condition $\int E\,dl = E_0 d$ must be satisfied, where d is the electrode separation.

When large currents flow the field distribution is much more severely distorted as shown in figure 12 (*b*). The field everywhere falls far below E_0 except in the vicinity of the cathode where it may become very large.

From Poisson's equation the field is given by

$$\frac{dE}{dx} = \frac{ne}{\epsilon_0} = \frac{I}{Av\epsilon_0},$$

where n is the ion number density, e the charge per particle, ϵ_0 the free space permittivity, I the total current, A the electrode area, and v the ion velocity.

This yields

$$\mu^+ E \frac{dE}{dx} = \frac{I}{A\epsilon_0}, \quad \text{or} \quad \frac{d}{dx}(\tfrac{1}{2}E^2) = \frac{I}{A\mu^+\epsilon_0}.$$

This gives on integration

$$[\tfrac{1}{2}E^2]_{E_{\max}}^{E_x} = \left[\frac{Ix}{A\mu^+\epsilon_0}\right]_0^x, \quad \text{or} \quad E_{\max}{}^2 - E_x^2 = \frac{2Ix}{A\mu^+\epsilon_0},$$

where x lies between 0 and d.

If x_m denotes that position where E becomes negligible then it follows (from $\int E\,dl = E_0 d$) that $\frac{1}{2}E_{\max} x_m = E_0 d$, to a close approximation, and

$$E_{\max}{}^2 = \frac{2I}{A\mu^+\epsilon_0} \frac{2E_0 d}{E_{\max}} \quad \text{or} \quad E_{\max}{}^3 = \frac{4IE_0 d}{A\mu^+\epsilon_0}.$$

Inserting the appropriate values for the present experiment of $E_0 d = 200\,\text{V}$, $I = 10{,}000$ amp and $A = 28\,\text{cm}^2$ gives $E_{\max} \doteqdot 10^5$ V per cm. This field is sufficient to accelerate ions to a high velocity and, as they strike the cathode, electron emission will follow. For argon the ratio of electron mobility to ion mobility is about 300 and it may be shown that the ion velocity at the cathode is an order of magnitude larger than the electron velocity at the anode. Thus, when large currents are drawn, the electron mobility is the controlling mechanism and the electron conductivity of the plasma is the significant factor.

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REFERENCES

Brogan, T. R. 1956 Thesis, Cornell University. The Conduction of Electric Current to Cold Electrodes in Shock Tubes.

de Leeuw, J. H. 1958 *University of Toronto Institute of Aerophysics Report*, no. 49.

Lin, S. C., Resler, E. L. & Kantrowitz, A. 1955 *J. Appl. Phys.* **26**, 95.

Neuringer, J. L. 1960 *J. Fluid Mech.* **7**, 287.

Pain, H. J. & Smy, P. R. 1960*a* *Proc. Phys. Soc.* **76**, 849.

Pain, H. J. & Smy, P. R. 1960*b* *J. Fluid Mech.* **9**, 390.

Patrick, R. M. & Brogan, T. 1959 *J. Fluid Mech.* **5**, 289.

Rosa, R. J. 1960 *J. Appl. Phys.* **31**, 735.

Sakuntala, M., von Engel, A. & Fowler, R. G. 1960 *Phys. Rev.* **118**, 1459.